Robust Positioning with Single Frequency Inertially Aided RTK

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BIOGRAPHY

Dr. Bruno M. Scherzinger obtained the B.Eng. degree from McGill University in 1977 and the M.A.Sc. and Ph.D. degrees in system control engineering from the University of Toronto respectively in 1979 and 1983. He is a founding partner and Chief Scientist at Applanix Corporation. He has recently been appointed an Adjunct Professor at the University of Calgary's Geomatics Engineering Department.

ABSTRACT

This paper describes an inertially aided RTK (IARTK) positioning system using single frequency GPS receivers. The test platform is a Position and Orientation System for Land Vehicles (POS/LV) under development at Applanix, whose design objectives are to achieve robust positioning with decimeter-level accuracy during various levels of satellite coverage ranging from total outage to complete coverage. The single frequency tests described here are intended to demonstrate the significant improvement in ambiguity resolution that IARTK is capable of.

This paper first describes the IARTK architecture implemented in the POS/LV. The paper then describes the IARTK experiment with only L1 observables. The experiment comprised a series of tests of RTK performance over baselines up to 6 kilometers. The test results show that the IARTK algorithm requires between 90 seconds and 10 minutes to establish an initial L1 integer ambiguity resolution, as would be expected. Thereafter the IARTK algorithm recovers L1 integer ambiguities within 10 seconds following outages up to 30 seconds. These results show that IARTK can provide rapid L1 ambiguity resolution over baselines out to 5 kilometers. They demonstrate the significant difference that inertial aiding makes on time to fix ambiguities through the reduction of search space volume.

INTRODUCTION

This paper presents an inertially aided real-time kinematic (IARTK) system that normally operates with dual frequency observables to achieve rapid ambiguity resolution, and then demonstrates the performance of the IARTK system with only single frequency observables. The work reported here is a continuation of that reported by the author in [3] and [4]. The purposes of this experiment are twofold. The first is to demonstrate the significant RTK performance improvement that inertial aiding can provide. Without any a priori precise position information, the time to fix L1 integer ambiguities following a GPS outage using only L1 pseudorange and phase observables is typically on the order of several minutes on short baselines on the order of a few kilometers and is often simply not achievable on longer baselines, typically beyond a few kilometers. L1-only ambiguity resolution is sensitive to residual atmospheric and multipath errors in the double-differenced observables and to the a priori position uncertainty that defines the initial search space volume. This sensitivity makes L1-only RTK impractical for precise positioning, which is evidenced by the dominance of dual frequency RTK-capable receivers. This severe limitation makes L1-only RTK an interesting candidate for examining the performance improvement that inertial aiding can provide. The second purpose of this work is to assess the feasibility and practicality of a reliable robust positioning IARTK system using single frequency receivers. To date, single frequency receivers that deliver good quality observables continue to cost significantly less than dual frequency receivers.

Robust positioning describes a positioning system's ability to maintain position data continuity and accuracy through most or all anticipated operational conditions. The operational condition of interest here is the loss of good GPS satellite visibility due to partial or total obstruction of the sky. The key performance attribute investigated here is the time to re-acquire integer RTK after full GPS outages. In a dual-frequency RTK system, the time duration of an outage of observables from a particular satellite is the outage time of dual frequency data. Hence the outage time comprises the time of actual signal shading plus the receiver's time to reacquire full dual-frequency phase lock on the satellite signal. Some receivers will re-acquire L1 phase up to 10 seconds before re-acquiring L2 phase. Consequently the ability to rapidly fix L1 integer ambiguities without L2 data after L1 phase re-acquisition becomes advantageous.

The Position and Orientation System for Land Vehicles (POS/LV) described here is a tightly coupled inertial/GPS integrated system with tightly coupled RTK shown in Figure 1. *Tightly coupled inertial/GPS* integration implies the Kalman filter processes the GPS pseudorange, phase and Doppler observables. *Tightly coupled RTK* implies the Kalman filter that estimates the inertial navigator errors also estimates the floated phase ambiguities, and an on-the-fly (OTF) ambiguity resolution algorithm operates on these to fix the integer ambiguities. In this case, the GPS receiver is strictly a sensor of the GPS observables. Any GPS receiver that outputs the observables and satellite ephemeredes can be used. The navigation functions in the GPS receiver, namely position and clock offset fixing and the receiver's RTK function, are not used.

SYSTEM DESCRIPTION

Figure 1 shows the architecture of an IARTK system, comprising a standard closed-loop aided INS configuration with a few non-standard components. The sensor components comprise the inertial measurement unit (IMU), roving GPS receiver and base receiver, and a precise odometer here called a distance measurement indicator (DMI). The IMU provides measurements of incremental velocities and angles resolved in the IMU sensor coordinate frame. The roving receiver provides observables and ephemeredes for the visible satellites. Normally these would be dual frequency; for this experiment they are restricted to single frequency observables. The base receiver provides standard RTK messages in RTCM or CMR format that contain the base receiver observables and clock error. The DMI measures the rotations of an instrumented wheel and from this reports the distance of the wheel ground track.

The processing components comprise the inertial navigator, Kalman filter, error controller and OTF ambiguity resolution module. The inertial navigator computes the position, velocity and orientation of the IMU sensor frame from the IMU data. The Kalman filter estimates the errors in the inertial navigator, IMU, DMI and GPS receivers. It implements the following states:

- inertial navigator errors (see [1]),
- gyro and accelerometer biases,
- DMI errors,
- floated phase DD ambiguities.

It constructs the following measurements:

- inertial-GPS DD pseudoranges
- inertial-GPS DD L1 phases
- inertial-DMI integrated speed (optional)

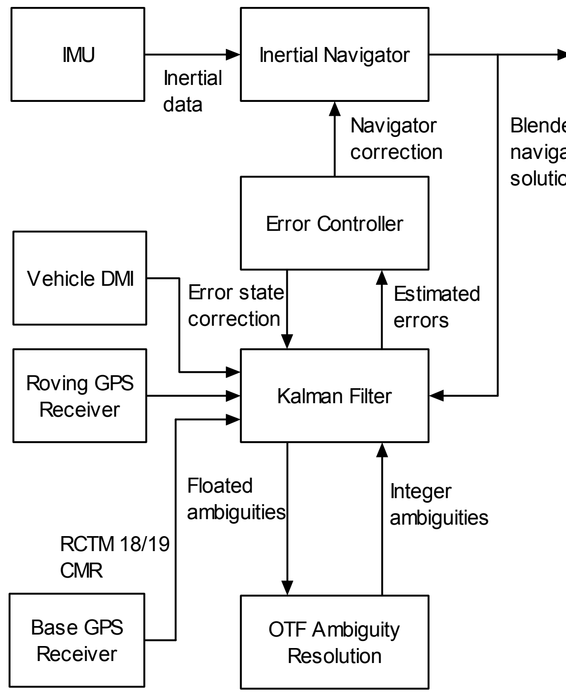

Figure 1: IARTK system architecture

where DD implies double differences of the base and roving receiver observables. Additional components not shown can include a GPS azimuth measurement system (GAMS), which is part of the Applanix POS/LV. The GAMS provides direct heading observations, which provide for latitude-independent regulation of the inertial navigator heading error.

The j-th double-differenced phase measurement for a single baseline is given as follows:

$$z_{\phi_j} = \nabla\Delta r_j\left(\hat{\vec{r}}_{SNV}, \vec{r}_{base}\right) + \lambda\left(\nabla\Delta\phi_j - \nabla\Delta N_{j0}\right) \qquad (1)$$

where:

λ	is the widelane or L1 wavelength,
$\hat{\vec{r}}_{SNV}$	is the computed inertial navigator position,
$\vec{r}_{base}$	is the base receiver position,
$\nabla\Delta\phi_j$	is the double-differenced phase,
$\nabla\Delta N_{j0}$	is the initial integer ambiguity,

$$\nabla\Delta r_j\left(\hat{\vec{r}}_{SNV},\vec{r}_{base}\right)=\Delta r_j\left(\hat{\vec{r}}_{SNV},\vec{r}_{base}\right)-\Delta r_b\left(\hat{\vec{r}}_{SNV},\vec{r}_{base}\right)$$

is the predicted range double difference to the j-th and base satellites in a base satellite double-differencing method,

$$\Delta r_j\left(\hat{\vec{r}}_{SNV},\vec{r}_{base}\right)=r_j\left(\hat{\vec{r}}_{SNV}\right)-r_j\left(\vec{r}_{base}\right)$$

is the single difference between the inertial navigator position and the base receiver position for the j-th satellite, and

$$r_j\left(\hat{\vec{r}}_{SNV}\right)=\left|\vec{r}_{SV_j}-\hat{\vec{r}}_{SNV}\right|$$

is the range from the inertial navigator position to the jth satellite. The measurement model is given as follows:

$$z_{\phi_j}=-\left(\vec{e}_j-\vec{e}_b\right)\cdot\delta\vec{r}+\lambda a_j+\eta_{\phi_j} \tag{2}$$

where:

- $\delta\vec{r}$ is the inertial navigator position error in the modified ψ-angle inertial navigator error model [1],
- a_j is the ambiguity double-difference,
- η_{ϕ_j} is the phase double-difference noise,
- $\vec{e}_j$ is the unit line-of-sight (LOS) vector from the inertial position to the j-th satellite, given by:

$$\vec{e}_j\cong\frac{\partial r_j\left(\hat{\vec{r}}_{SNV}\right)}{\partial\hat{\vec{r}}_{SNV}}=\frac{1}{r_j\left(\hat{\vec{r}}_{SNV}\right)}\begin{bmatrix}x_{SV_j}-\hat{x}_{SNV}\\ y_{SV_j}-\hat{y}_{SNV}\\ z_{SV_j}-\hat{z}_{SNV}\end{bmatrix} \tag{3}$$

These observables measurements implement what is often referred to as ”tightly coupled” inertial/GPS integration. The error controller computes and applies corrections to the inertial navigator from the estimated navigation errors generated by the Kalman filter. It closes the error control loop that regulates the inertial navigator errors to be consistent with those of the aiding sensors, in this case the GPS receiver. The blended navigation solution exhibits the combined attributes of position solution continuity from the inertial navigator and position accuracy brought about by the error regulation.

The OTF ambiguity resolution module operates on the floated phase DD ambiguities sub-state and covariance sub-matrix, and attempts to fix the corresponding integer ambiguities. It implements a modified Fast Ambiguity Search Filter (FASF) algorithm first proposed by Chen [5]. This algorithm is particularly well suited for integration with the inertial/GPS Kalman filter. It performs the integer search in the ambiguity space defined by the floated ambiguity states in the Kalman filter. The modification to the original algorithm reported in Figure 1 is the inclusion of an ambiguity decorrelation algorithm similar to that reported in [6].

INERTIAL AIDING OF RTK

Inertial aiding of ambiguity resolution is a consequence of a single Kalman filter for estimating both inertial navigation errors and floated ambiguities. A single stochastic model describes both the inertial navigator errors and the floated ambiguities. The ambiguity search algorithm operates on a sub-vector $\vec{a}$ of the estimated error state vector contained the floated ambiguities and a sub-matrix P_a of the estimation error VCV matrix comprising the VCV matrix of the floated ambiguities. This method of integration is called *tightly-coupled ambiguity resolution*, and provides for enhanced observability of the floated ambiguities when the inertial position error is sufficiently small via the cross-correlation between roving position error and floated ambiguities that the phase DD measurement model (2) generates. The information matrix characterizes observability in stochastic estimation, which is the inverse of the estimation error VCV matrix (see Jazwinski [10] for an elaboration of stochastic observability). Here the information sub-matrix P_a^{-1} characterizes the observability of the floated ambiguities. Equivalently, the ambiguity search space volume is made smaller with the extrapolation of the baseline vector that an INS or any other form of DR navigation provides. (see Skaloud [7]). The volume of the search space is proportional to the ambiguity dilution of precision (ADOP) introduced by Teunissen et al in [9] and given as follows:

$$ADOP=\left(\sqrt{\det\left(P_a\right)}\right)^{\frac{1}{n}} \tag{4}$$

ADOP is given in units of cycles, and is invariant under volume-preserving transformations of the ambiguities such as ambiguity decorrelation transformations. Teunissen [9] describes ADOP as a measure of the precision of the floated ambiguities. Enhanced observability of the floated ambiguities from the a priori INS position error is thus equivalent to retained ambiguity precision from the inertial navigator position error and to a reduction in the search space volume. The following covariance analysis from [4] supports this observation. References to phase measurements and ambiguities should here be interpreted as coming from L1 phase data processing, although the covariance analysis is applicable to any phase or integer combination phases used for ambiguity resolution.

The IARTK Kalman filter state can be partitioned as follows:

$$\vec{x} = \begin{bmatrix} \vec{x}_s \\ \vec{a} \end{bmatrix} \qquad \vec{x}_s = \begin{bmatrix} \delta\vec{r} \\ \vec{x}_2 \end{bmatrix} \tag{5}$$

where $\vec{x}_s$ is the inertial navigation error state with dimension n_s, containing inertial position error $\delta\vec{r}$ and the remaining n_s-3 states $\vec{x}_2$, typically velocity error, misalignment error and inertial sensor errors. $\vec{a}$ is the vector of phase ambiguities in the double-differenced phase measurements (2) with dimension m-1 where m is the number of pseudorange and phase observables. The pseudorange and phase measurements are written in vector form as follows:

$$\vec{z}_\rho = D^T A\delta\vec{r}_s + \vec{\eta}_\rho = \left[H_s \;\vdots\; 0 \right]\vec{x} + \vec{\eta}_\rho \tag{6}$$

$$\vec{z}_\phi = D^T A\delta\vec{r}_s + \lambda\vec{a} + \vec{\eta}_\phi = \left[H_s \;\vdots\; H_a \right]\vec{x} + \vec{\eta}_\phi \tag{7}$$

where $$H_s = \left[D^T A \;\vdots\; 0_{(m-1)\times(n_s-3)} \right] \tag{8}$$

A is the single-difference measurement model matrix and D^T is the $(m$-1$)\times m$ double difference operator, both described in [6]. $H_a = \lambda I$ where λ is the phase wavelength. The pseudorange and phase measurement noises $\vec{\eta}_\rho$ and $\vec{\eta}_\phi$ are uncorrelated across epochs and have respective VCV matrices R_ρ and R_ϕ.

We want to investigate the propagation of the floated ambiguity estimate from the IARTK Kalman filter after the first phase measurement (7) following a GPS outage that causes a discontinuity in the phase data. The Kalman filter a posteriori VCV matrix is given by:

$$P^+ = P^- - P^- H^T \left(HP^- H^T + R \right)^{-1} HP^- \tag{9}$$

The a priori and a posteriori VCV matrices are partitioned compatible with (5) as follows:

$$P = \begin{bmatrix} P_s & P_{sa} \\ P_{sa}^T & P_a \end{bmatrix} \quad P_s = \begin{bmatrix} P_{\delta r} & P_{12} \\ P_{12}^T & P_2 \end{bmatrix} \tag{10}$$

where $P_{\delta r}$ is the estimation error VCV matrix for $\delta\vec{r}$. The floated ambiguity states are re-initialized following a GPS outage to reflect complete loss of phase information, so that

$$P_a^- = \sigma_{a_0}^2 D^T D \;\text{ and }\; P_{sa}^- = 0 \tag{11}$$

The VCV matrix for the updated floated ambiguity estimates is obtained from (8), (9) and (10) as follows:

$$P_a^+ = P_a^- - P_a^- H_a^T R_a^{-1} H_a P_a^- \tag{12}$$

where $$R_a = D^T A P_{\delta r}^- A^T D + H_a P_a^- H_a^T + R_\phi \tag{13}$$

The closed-loop architecture in Figure 1 causes the inertial navigator to be corrected by the estimated navigation errors and hence $P_{\delta r}^-$ to approximate the actual a priori inertial navigator position error VCV. The matrix norm $\left| P_{\delta r}^- \right| = trace\left(P_{\delta r}^- \right)$ gives the expected position error variance, which is the normal expression of position solution accuracy. We say that a position solution 1 is as accurate or more accurate than a position solution 2 if $P_{\delta r_2}^- - P_{\delta r_1}^-$ is positive semi-definite, which we represent as a matrix inequality $P_{\delta r_1}^- \le P_{\delta r_2}^-$. This is a more conservative method of representing position solution accuracy than with variances. We can rank the accuracies of the floated ambiguity VCV matrices associated with the two position error VCV matrices as follows:

$$P_{a_1}^+ - P_{a_2}^+ = P_a^- H_a^T \left(R_{a_2}^{-1} - R_{a_1}^{-1} \right) H_a P_a^- \tag{14}$$

$$= \lambda^2 \sigma_{a_0}^4 D^T D R_{a_2}^{-1} D^T A \left(P_{\delta r_1}^- - P_{\delta r_2}^- \right) A^T D R_{a_1}^{-1} D^T D$$

The right-hand-side of (14) is the product of positive semi-definite matrices and the quadratic form

$$D^T A \left(P_{\delta r_1}^- - P_{\delta r_2}^- \right) A^T D$$

This implies $P_{a_1}^+ \le P_{a_2}^+$ if and only if $P_{\delta r_1}^- \le P_{\delta r_2}^-$. This provides a qualitative description of observability enhancement and equivalent ambiguity search space volume reduction through IARTK. If two estimation methods generate different a priori position errors expressed by:

$$P_{\delta r_1}^- \le P_{\delta r_2}^- \tag{15}$$

then the observability enhancement obtained from the more accurate estimation method is expressed by:

$$P_{a_1}^{+^{-1}} \ge P_{a_2}^{+^{-1}} \tag{16}$$

and the search space volume reduction is expressed by:

$$ADOP_1 \le ADOP_2 \tag{17}$$

We use this analysis to characterize the performance improvement that IARTK provides over unaided RTK by declaring position solution 1 to be the inertial position at the end of an outage and position solution 2 to be a re-initialized position computed from double differenced pseudoranges without a previous inertial position. If the inertial position drift is small, then solution 1 will retain the decimeter level fixed integer RTK error with some drift through the outage, whereas solution 2 will exhibit meter level position errors that are typical of a code phase differential solution. The solution 1 search space volume will be correspondingly smaller than the solution 2 search space volume. The inequalities (16) and (17) will tend to equalities with increasing GPS outage duration and consequent increasing inertial position error, as was also noted by Skaloud [7].

Figure 2: POS/LV 320 used in IARTK tests

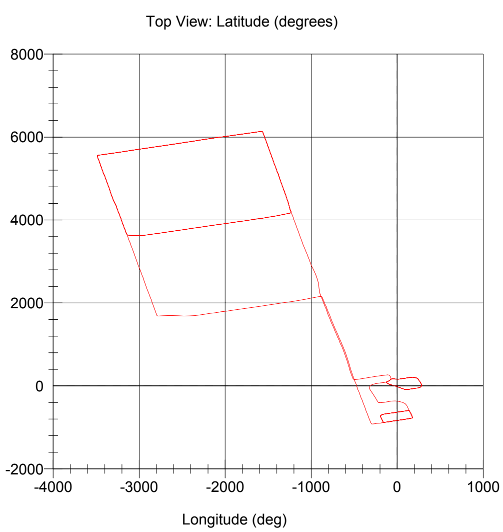

Figure 3: Test trajectory plan view

SINGLE FREQUENCY TEST RESULTS

This section describes the results of an investigation that explores the IARTK performance of the POS with L1 phase data using the following evaluation criteria:

- time to recover RTK accuracy after full outages,
- position error during a full GPS outage.

A full GPS outage is here defined to be the complete absence of any or all GPS observables data to the navigation-processing algorithm. It includes the time that a GPS antenna is completely shaded plus the receiver re-acquisition time.

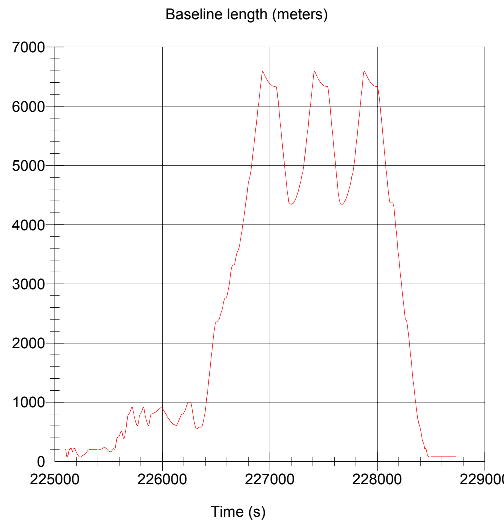

Figure 4: Baseline separation

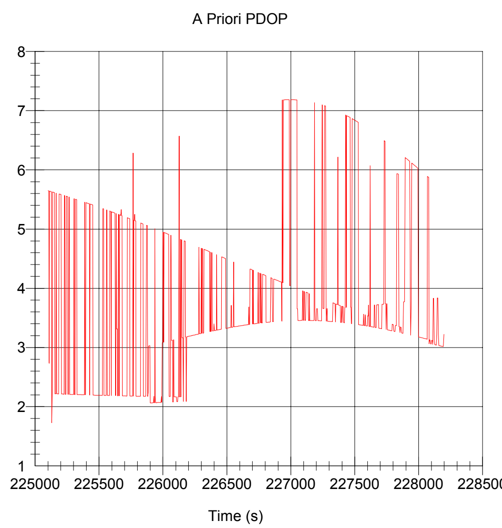

Figure 5: PDOP

This test used a modified POS/LV 320 (standard model is shown in Figure 2) to run the real-time solution and record the data, and the Real-Time Simulator (RTSIM) for the POS/LV embedded software to reproduce the real-time navigation processing and to introduce test events such as full or partial GPS outages. The POS/LV contained a single frequency Novatel 3151R GPS receiver and a tactical-grade IMU having the following performance attributes:

- 3 degrees/hour gyro bias
- 0.1 degrees/√hour gyro random walk
- 1500 μg accelerometer bias

RTSIM is an exact copy of the navigation-processing component of the POS embedded software, and includes simulated multiple concurrent processes, processing delays, inter-process messages and rendezvous. It includes the same RTCM and CMR decoders that the embedded system uses. The reference solution for error evaluation is a blended navigation solution computed by Applanix's POSPAC post-processing software [2]. One of the test trajectories in this test is shown in Figure 3 and its baseline separation between roving and base receivers is shown in Figure 4. Figure 5 shows the PDOP during the test. The satellite coverage ranges from good to poor with frequent fluctuations due to masking of critical satellites for good geometry by foliage along the trajectory. This is typical GPS coverage along tree-lined roads and a challenging environment for ambiguity resolution.

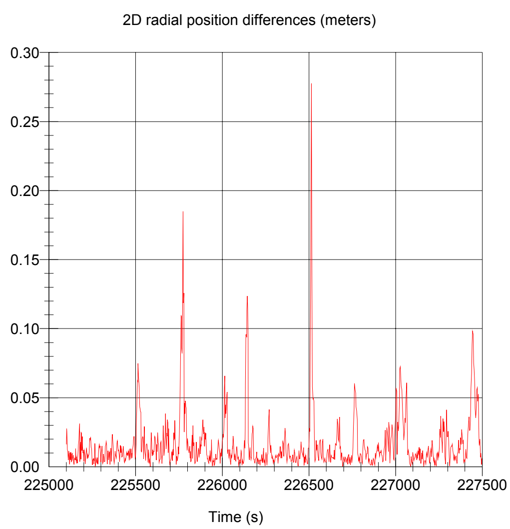

Figure 6: Radial position error with 10-second GPS outages

The time-to-fix for this test is the elapsed time between L1 phase lock and L1 integer ambiguity resolution. The method of testing the time-to-fix dependency on outage duration is to introduce artificial outages of increasing duration and note the time-to-fix. Figure 6 and Figure 8 show the radial position errors respectively for 10 and 30-second outages. Table 1 shows the position error standard deviations and times to L1 integer ambiguity resolution. The acronym *NR* designates no resolution before the start of the next scheduled outage. Reliable times-to-fix within 10 seconds were demonstrated following 10-second outages. Reliable times-to-fix within 30 seconds were demonstrated following up to 30-second outages. Thereafter ambiguity resolution within a reasonable wait time became unreliable.

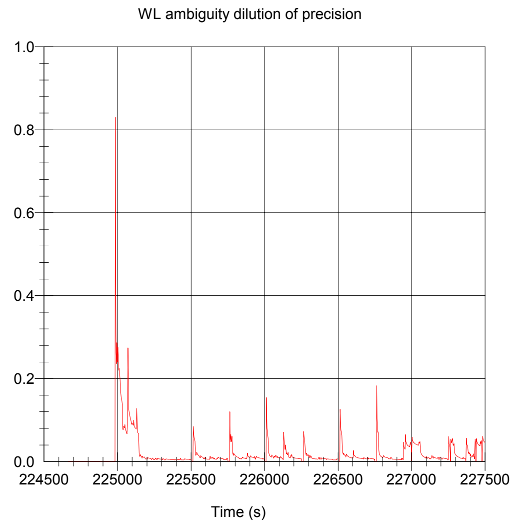

Figure 7: L1 ADOP with 10-second GPS outages

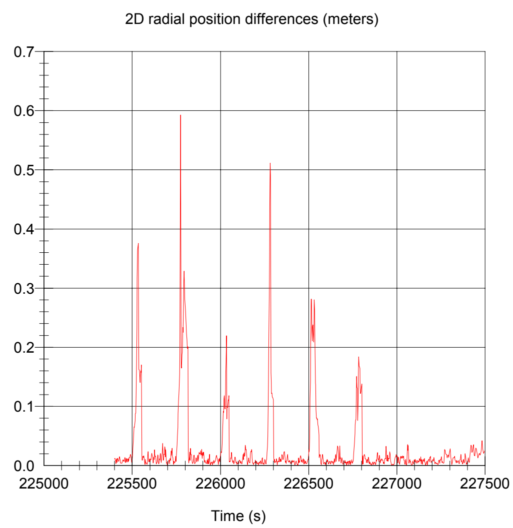

Figure 8: Radial position error with 30-second GPS outages

Table 1: Single frequency ambiguity resolution and positioning statistics

Outage Duration (seconds)	Radial Position Error RMS (meters)	Time-to-fix L1 ineger ambiguities (seconds)
10	0.1	5, 6, 4, 2, 5, 4
30	0.3	6, 22, 5, 7, 14, 8
60	0.8	NR, 11, 5, 7, 143, NR
120	1.4	NR, NR, NR, 71, 75, 19

Figure 7 shows the ADOP with 10-second GPS outages. The ADOP at the start of processing is large, and describes the search space volume before a first fixed integer RTK solution has been computed. Thereafter the ADOP and corresponding search space volume following an outage are significantly smaller, sufficiently so to allow rapid L1 integer fixing. Figure 7 exemplifies equation (17) of the previous covariance analysis, showing both an initial $ADOP_2$ without a precise a priori position solution plus a series of significantly smaller $ADOP_1$ following GPS outages that result from inertial aiding.

CONCLUSIONS

This paper has examined the performance of a position and orientation system with inertially aided RTK in an experiment that used only L1 observables. All practical GPS receivers with RTK capability use L1 and L2 phases to achieve integer ambiguity resolution within one to two minutes. Single frequency ambiguity resolution requires a significantly longer time, upwards of 5 minutes, which makes it impractical for kinematic positioning. This experiment was primarily intended to demonstrate the significant difference in performance and improvement in robustness that inertial aiding can provide to an ambiguity resolution process. It has shown that inertial aiding provides significant improvement in the time-to-fix and reliability of fixes over unaided RTK. Inertial aiding of RTK can provide a precise a priori position solution after a GPS outage, which allows the IARTK algorithm to fix L1 integer ambiguities in seconds and thereby achieve the same robust positioning characteristics as was described for a dual frequency IARTK system in [3] and [4] on baselines to 5 kilometers.

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