Robust Inertially-Aided RTK Position Measurement

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BIOGRAPHY

Dr. Bruno M. Scherzinger obtained the B.Eng. degree from McGill University in 1977 and the M.A.Sc. and Ph.D. degrees in system control engineering from the University of Toronto respectively in 1979 and 1983. He is a founding partner and Chief Scientist at Applanix Corporation.

ABSTRACT

This paper describes "robust" positioning as the ability to determine position with continuous accuracy in spite of GPS outages. The paper describes the Position and Orientation System (POS) with inertially-aided RTK (POS-IARTK) as one solution to the robust positioning problem. The concept and initial experimental test results were presented in [1]. The paper first describes the architecture of the POS-IARTK. It then reports on the results of a series of van tests that demonstrate the real-time performance of the POS-IARTK in urban canyons and foliage.

INTRODUCTION

This paper is concerned with the problem of "robust" positioning, defined here to describe the combined attributes of continuous position data with expected or required accuracy throughout most or all operational conditions. These include conditions that are hostile to GPS, in particular conditions that cause partial or complete blockage of GPS satellites.

GPS falls in the category of positioning systems that uses external sources of signal to compute its position solution either as a trilateration of ranges to known reference positions (i.e. satellites) or as measured polar coordinates from a single reference position (i.e. tracking radar or laser). Other examples of signal-based position systems include Loran C, VOR, microwave ranging and laser tracking systems. They have the common characteristics of approximately constant position accuracy and sensitivity to loss or distortion of their reference signals. They are dependent on external infrastructure and its ongoing normal operation. GPS has become the dominant positioning system among commercial users because it offers global accessibility and excellent positioning accuracy. However a GPS receiver's access to GPS signals can be interrupted by shading, jamming or ionospheric scintillation. Pseudolites may provide signal sources that are closer to the receiver than the satellites, thereby reducing but not eliminating potential outages. Such systems cannot be considered robust because of their sensitivity to potential signal blockages and distortions.

Dead reckoning (DR) navigation systems (DRNS) navigate by computing accumulated position change from a known position using self-contained dynamics sensors and integration of their measured quantities. A simple vehicle DR navigator uses an odometer and heading sensor to compute accumulated North and East position change. An inertial navigation system (INS) computes position from accelerations and angular rates that its inertial sensors measure. A DRNS provides a continuous navigation solution under all conditions, and hence has the desirable property of position solution continuity. Integration of the DR dynamic sensor errors however results in position errors that accumulate with time. The dominant position error characteristics of a free-inertial INS are a Schuler oscillation superimposed on an approximately constant position error rate.

A hybrid positioning system is a synergistic combination of a radio positioning system with a DRNS to achieve both position data continuity and some maintenance of position accuracy throughout. It uses the dead reckoning system to "coast" through outages of the radio positioning system, and calibrates the dead reckoning system when radio position solutions are available. The common example is the GPS-aided INS. The design goal of hybrid system design is to exhibit the data continuity of an INS with the position accuracy of a GPS receiver, which is ideal robust positioning.

The Position and Orientation System (POS) with inertially-aided RTK (IARTK) is one solution to the robust positioning problem. The objective of IARTK is to exhibit ongoing robust positioning in real time with decimeter accuracy in a variety of environments that are normally hostile to GPS reception. The IARTK POS is required to maintain position accuracy with slow degradation following a GPS outage, and to recover integer RTK positioning rapidly when GPS data sufficient for positioning is again available. The goal is to approach ideal robustness under practical navigation conditions for applications such as land vehicle survey and machine control. The concept and initial experimental test results were presented in [1] and are reviewed here.

SYSTEM DESCRIPTION

Figure 1 shows the architecture of an IARTK system for land vehicles. The sensor components comprise the inertial measurement unit (IMU), roving GPS receiver and base receiver, and a precise odometer here called a distance measurement indicator (DMI). The IMU provides measurements of incremental velocities and angles resolved in the IMU sensor axis frame. The roving receiver provides dual-frequency observables and ephemeredes for the visible satellites. The base receiver provides standard RTK messages in RTCM or CMR format that contain the base receiver observables and clock error. The DMI measures the rotations of an instrumented wheel and from this reports the distance of the wheel ground track.

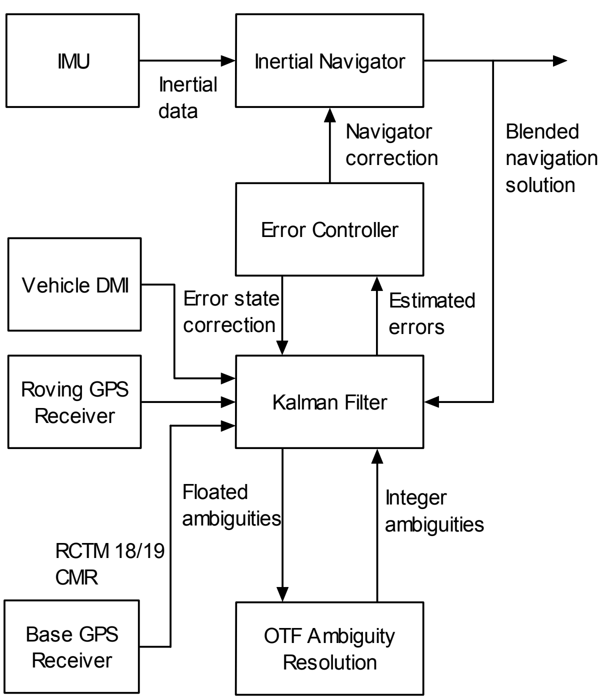

Figure 1: IARTK system architecture

The processing components comprise the inertial navigator, Kalman filter, error controller and OTF ambiguity resolution module. The inertial navigator computes the position, velocity and orientation of the IMU sensor frame from the IMU data. The Kalman filter estimates the errors in the inertial navigator, IMU, DMI and GPS receivers. It implements the following states:

- inertial navigator errors (see [2]),
- gyro and accelerometer biases,
- DMI scale factor and alignment errors,
- floated phase DD ambiguities.

It constructs the following measurements:

- inertial-GPS DD pseudoranges
- inertial-GPS DD L1 phases
- inertial-GPS DD wide-lane (L1-L2) phases
- inertial-DMI integrated speed

where DD implies double differences of the base and roving receiver observables. Additional components not shown can include a GPS azimuth measurement system (GAMS), which is part of the Applanix POS/LV. The GAMS provides direct heading observations, which provide for latitude-independent regulation of the inertial navigator heading error.

The *j*-th double-differenced phase measurement for a single baseline is given as follows:

$$z_{\phi_j} = \nabla\Delta r_j\left(\hat{\vec{r}}_{SNV}, \vec{r}_{base}\right) + \lambda\left(\nabla\Delta\phi_j - \nabla\Delta N_{j0}\right) \tag{1}$$

where:

λ	is the widelane or L1 wavelength,
$\hat{\vec{r}}_{SNV}$	is the computed inertial navigator position,
$\vec{r}_{base}$	is the base receiver position,
$\nabla\Delta\phi_j$	is the double-differenced phase,
$\nabla\Delta N_{j0}$	is the initial integer ambiguity,

$$\nabla\Delta r_j\left(\hat{\vec{r}}_{SNV}, \vec{r}_{base}\right) = \Delta r_j\left(\hat{\vec{r}}_{SNV}, \vec{r}_{base}\right) - \Delta r_b\left(\hat{\vec{r}}_{SNV}, \vec{r}_{base}\right)$$

is the predicted range double difference to the *j*-th and base satellites in a base satellite double-differencing method,

$$\Delta r_j\left(\hat{\vec{r}}_{SNV}, \vec{r}_{base}\right) = r_j\left(\hat{\vec{r}}_{SNV}\right) - r_j\left(\vec{r}_{base}\right)$$

is the single difference between the inertial navigator position and the base receiver position for the *j*-th satellite, and

$$r_j\left(\hat{\bar{r}}_{SNV}\right) = \left|\bar{r}_{SV_j} - \hat{\bar{r}}_{SNV}\right|$$

is the range from the inertial navigator position to the *j*th satellite. The measurement model is given as follows:

$$z_{\phi_j} = -\left(\bar{e}_j - \bar{e}_b\right)\cdot\delta\bar{r} + \lambda a_j + \eta_{\phi_j} \tag{2}$$

where:

- $\delta\bar{r}$ is the inertial navigator position error in the modified ψ-angle inertial navigator error model [2],
- a_j is the ambiguity double-difference,
- η_{ϕ_j} is the phase double-difference noise,
- $\bar{e}_j$ is the unit line-of-sight (LOS) vector from the inertial position to the *j*-th satellite, given by:

$$\bar{e}_j \cong \frac{\partial r_j\left(\hat{\bar{r}}_{SNV}\right)}{\partial\hat{\bar{r}}_{SNV}} = \frac{1}{r_j\left(\hat{\bar{r}}_{SNV}\right)}\begin{bmatrix} x_{SV_j} - \hat{x}_{SNV} \\ y_{SV_j} - \hat{y}_{SNV} \\ z_{SV_j} - \hat{z}_{SNV} \end{bmatrix} \tag{3}$$

These observables measurements implement the "tightly coupled" inertial/GPS integration. It has notable advantages over a loosely coupled integration, in particular providing ongoing aiding to the inertial navigator when the number of visible satellites drops below the minimum 4 needed to compute a GPS position fix. Tightly coupled inertial/GPS integration not new, however it does provide the measurements and models for inertially aided RTK, the main topic of this paper.

The error controller computes and applies corrections to the inertial navigator from the estimated navigation errors generated by the Kalman filter. It closes the error control loop that regulates the inertial navigator errors to be consistent with those of the aiding sensors, in this case the GPS receiver. The blended navigation solution exhibits the combined attributes of position solution continuity from the inertial navigator and position accuracy brought about by the error regulation. This method provides continuous alignment of the inertial navigator while GPS aiding data are available, and thereby provides an accurate orientation solution.

The OTF ambiguity resolution module operates on the floated phase DD ambiguities sub-state and covariance sub-matrix, and attempts to fix the corresponding integer ambiguities. It implements a modified Fast Ambiguity Search Filter (FASF) algorithm first proposed in [3]. This algorithm is particularly well suited for integration with the inertial/GPS Kalman filter. It performs the integer search in the ambiguity space defined by the floated ambiguity states in the Kalman filter. The modification to the original algorithm reported in [3] is the inclusion of an ambiguity decorrelation algorithm similar to that reported in [4].

INERTIAL AIDING OF RTK

Inertial aiding of ambiguity resolution is a consequence of a single Kalman filter for estimating both inertial navigation errors and floated ambiguities. A single stochastic model describes both the inertial navigator errors and the floated ambiguities. The ambiguity search algorithm operates on a sub-vector $\bar{a}$ of the estimated error state vector contained the floated ambiguities and a sub-matrix P_a of the estimation error VCV matrix comprising the VCV matrix of the floated ambiguities. This method of integration is called *tightly-coupled ambiguity resolution*, and provides for enhanced observability of the floated ambiguities when the inertial position error is sufficiently small via the cross-correlation between roving position error and floated ambiguities that the phase DD measurement model (2) generates. An alternative interpretation was discussed and developed in [6], stating that the ambiguity search space volume is made smaller with the extrapolation of the baseline vector that an INS or any other form of DR navigation provides. These two interpretations are in fact the same. The information matrix characterizes observability in stochastic estimation, which is the inverse of the estimation error VCV matrix [8]. In this case the information sub-matrix P_a^{-1} characterizes the observability of the floated ambiguities. The volume of the search space is proportional to the ambiguity dilution of precision (ADOP) introduced in [7] and given as follows:

$$ADOP = \left(\sqrt{\det\left(P_a\right)}\right)^{\frac{1}{n}} \tag{4}$$

ADOP is given in units of cycles, and is invariant under volume-preserving transformations of the ambiguities such as ambiguity decorrelation transformations. [7] describes ADOP as a measure of the precision of the floated ambiguities. Enhanced observability of the floated ambiguities from the a priori INS position error is thus equivalent to retained ambiguity precision from the inertial navigator position error and to a reduction in the search space volume. The following analysis formalizes this observation.

The IARTK Kalman filter state can be partitioned as follows:

$$\vec{x} = \begin{bmatrix} \vec{x}_s \\ \vec{a} \end{bmatrix} \qquad \vec{x}_s = \begin{bmatrix} \delta\vec{r} \\ \vec{x}_2 \end{bmatrix} \tag{5}$$

where $\vec{x}_s$ is the inertial navigation error state with dimension n_s, containing inertial position error $\delta\vec{r}$ and the remaining n_s-3 states $\vec{x}_2$, typically velocity error, misalignment error and inertial sensor errors. $\vec{a}$ is the vector of phase ambiguities in the double-differenced phase measurements (2) with dimension m-1 where m is the number of pseudorange and phase observables. The pseudorange and phase measurements are written in vector form as follows:

$$\vec{z}_\rho = D^T A\delta\vec{r}_s + \vec{\eta}_\rho = \left[H_s \;\vdots\; 0 \right]\vec{x} + \vec{\eta}_\rho \tag{6}$$

$$\vec{z}_\phi = D^T A\delta\vec{r}_s + \lambda\vec{a} + \vec{\eta}_\phi = \left[H_s \;\vdots\; H_a \right]\vec{x} + \vec{\eta}_\phi \tag{7}$$

where $$H_s = \left[D^T A \;\vdots\; 0_{(m-1)\times(n_s-3)} \right] \tag{8}$$

A is the single-difference measurement model matrix and D^T is the $(m\text{-}1)\times m$ double difference operator, both described in [5]. $H_a = \lambda I$ where λ is the phase wavelength. The pseudorange and phase measurement noises $\vec{\eta}_\rho$ and $\vec{\eta}_\phi$ are uncorrelated across epochs and have respective VCV matrices R_ρ and R_ϕ.

We want to investigate the propagation of the floated ambiguity estimate from the IARTK Kalman filter after the first phase measurement update (7) following a GPS outage that causes discontinuity in the phase data. The Kalman filter a posteriori VCV matrix is given by:

$$P^+ = P^- - P^- H^T \left(HP^- H^T + R \right)^{-1} HP^- \tag{9}$$

The a priori and a posteriori VCV matrices are partitioned compatible with (5) as follows:

$$P = \begin{bmatrix} P_s & P_{sa} \\ P_{sa}^T & P_a \end{bmatrix} \qquad P_s = \begin{bmatrix} P_{\delta r} & P_{12} \\ P_{12}^T & P_2 \end{bmatrix} \tag{10}$$

where $P_{\delta r}$ is the estimation error VCV matrix for $\delta\vec{r}$. The floated ambiguity states are re-initialized following a GPS outage to reflect complete loss of phase information, so that

$$P_a^- = \sigma_{a_0}^2 D^T D \quad \text{and} \quad P_{sa}^- = 0 \tag{11}$$

The VCV matrix for the updated floated ambiguity estimates is obtained from (8), (9) and (10) as follows:

$$P_a^+ = P_a^- - P_a^- H_a^T R_a^{-1} H_a P_a^- \tag{12}$$

where $$R_a = D^T A P_{\delta r}^- A^T D + H_a P_a^- H_a^T + R_\phi \tag{13}$$

The closed-loop architecture in Figure 1 causes the inertial navigator to be corrected by the estimated navigation errors and hence $P_{\delta r}^-$ to approximate the actual a priori inertial navigator position error VCV. The following are possible methods of estimating the inertial navigator position error during and after the GPS outage.

1. The IARTK Kalman filter processes DMI distance measurements during the GPS outage to estimate the inertial navigator position error with relatively low position drift.

2. The IARTK Kalman filter extrapolates the unaided or free-inertial navigation errors from the last GPS measurement. This describes the position error during free-inertial "coasting" through GPS outages.

3. A least-squares adjustment of the pseudorange measurement (6) as described in [5] generates the estimated position with typical pseudorange position accuracy with no a priori position information. This describes the position error following initialization of the IARTK or the limit of position accuracy with increasing GPS outage duration.

These methods of a priori position error estimation generate different navigation error VCV matrices P_s^- whose respective magnitudes can be ordered as follows. If we expect Method 1 from the above list to generate a smaller a priori navigation error that Method 2, then we say that we expect $P_{s_1}^- \le P_{s_2}^-$ which implies $P_{s_2}^- - P_{s_1}^-$ is positive semi-definite. Using this approach, we can determine the order of the resulting floated ambiguity VCV matrices as follows:

$$P_{a_1}^+ - P_{a_2}^+ = P_a^- H_a^T \left(R_{a_2}^{-1} - R_{a_1}^{-1} \right) H_a P_a^- \tag{14}$$

$$= \lambda^2 \sigma_{a_0}^4 D^T D R_{a_2}^{-1} D^T A \left(P_{\delta r_1}^- - P_{\delta r_2}^- \right) A^T D R_{a_1}^{-1} D^T D$$

The right-hand-side of (14) is the product of positive semi-definite matrices and the quadratic form

$$D^T A \left(P_{\delta r_1}^- - P_{\delta r_2}^- \right) A^T D$$

This implies $P^+_{a_1} \le P^+_{a_2}$ if and only if $P^-_{\delta r_1} \le P^-_{\delta r_2}$. This provides a qualitative description of observability enhancement and equivalent ambiguity search space volume reduction through IARTK. If the three estimation methods generate decreasing a priori position error expressed by:

$$P^-_{\delta r_1} \le P^-_{\delta r_2} \le P^-_{\delta r_3} \tag{15}$$

then the observability enhancement is expressed by:

$$P^{+^{-1}}_{a_1} \ge P^{+^{-1}}_{a_2} \ge P^{+^{-1}}_{a_3} \tag{16}$$

and the search space volume reduction is expressed by:

$$ADOP_1 \le ADOP_2 \le ADOP_3 \tag{17}$$

The above inequalities will tend to equalities with increasing GPS outage duration and consequent increasing inertial position error, as is also noted in [6].

DUAL FREQUENCY TESTS

This series of test was concerned with evaluating the performance of the POS-IARTK with dual frequency receivers. The performance attributes being tested are:

- Time-to-fix following L1 and L2 phase recovery and its dependency on outage duration,
- Preservation of position accuracy during GPS outages with and without additional DR aiding from the DMI.

The POS-IARTK uses a tactical grade IMU having the following inertial sensor characteristics:

- Accelerometer biases: 1500 µg
- Gyro biases: 3 degrees/hour
- Gyro random walk: 0.1 degrees/√hour

A characteristic of the GPS receiver is its L1 and L2 re-acquisition times following a GPS outage. Some receivers will acquire the L1 phase first and the L2 phase several seconds later. A dual-frequency based ambiguity resolution algorithm will begin to operate when the L2 phase data arrives, and hence exhibit an apparent penalty in time-to-fix from the time of L1 phase lock. The POS-IARTK used the Novatel Millenium receiver for this test, which re-acquires L1 and L2 phases at approximately the same time following an outage.

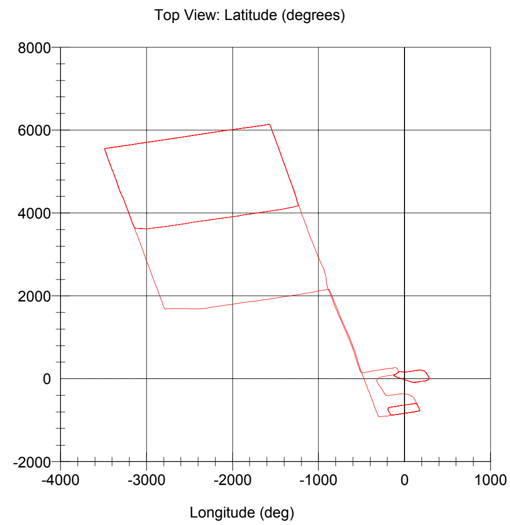

Figure 2: Test trajectory plan view

This test used a modified POS/LV 320 (standard model is shown in Figure 9) to run the real-time solution and record the data, and the Real-Time Simulator (RTSIM) for the POS embedded software to reproduce the real-time navigation processing and to introduce test events such as full or partial GPS outages. RTSIM is an exact copy of the navigation-processing component of the POS embedded software, and includes simulated multiple concurrent processes, processing delays, inter-process messages and rendezvous. It includes the same RTCM and CMR decoders that the embedded system uses.

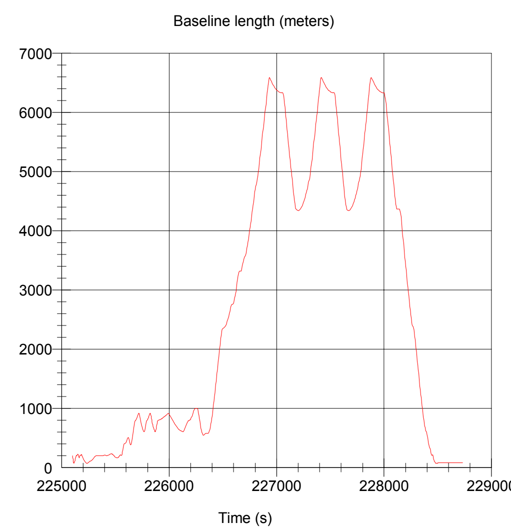

Figure 3: Baseline separation

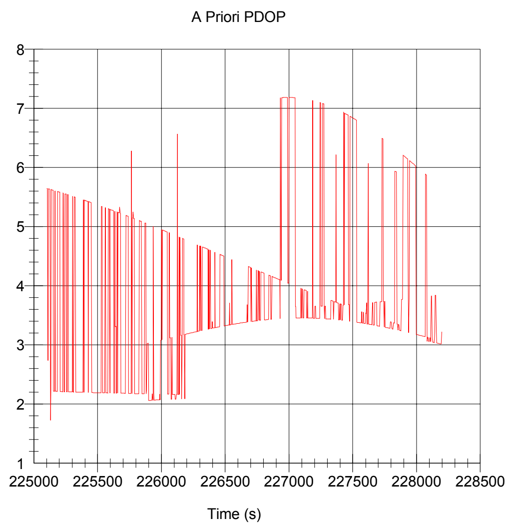

Figure 4: PDOP

The time-to-fix for this test is the elapsed time between L1 phase lock and L1 integer ambiguity resolution. The method of testing the time-to-fix dependency on outage duration is to introduce artificial outages of increasing duration and note the time-to-fix. The time-to-fix will be influenced by a number of factors such as phase noise and PDOP, and hence will exhibit a random component.

The test trajectory is shown in Figure 2 and its baseline separation between roving and base receivers is shown in Figure 3. Figure 4 shows the PDOP during the test. The satellite coverage ranges from good to poor with frequent fluctuations due to masking of critical satellites for good geometry by foliage along the trajectory. This is a challenging environment for ambiguity resolution.

Figure 5 and Figure 7 show the radial position errors respectively for 10 and 300-second outages. Table 1 lists the position error standard deviations and times to L1 integer ambiguity resolution following outages. These show a fairly reliable time to L1 ambiguity resolution within 10 seconds of L1 and L2 phase lock following outage durations to 120 seconds and possibly beyond. At 180 seconds a time-to-fix within 10 seconds can no longer be reliably achieved. Here and during larger scheduled outages there occur natural outages caused by the roadside foliage, which cause longer times-to-fix. At 600 seconds the time-to-fix after an outage is essentially the same as an initial time-to-fix.

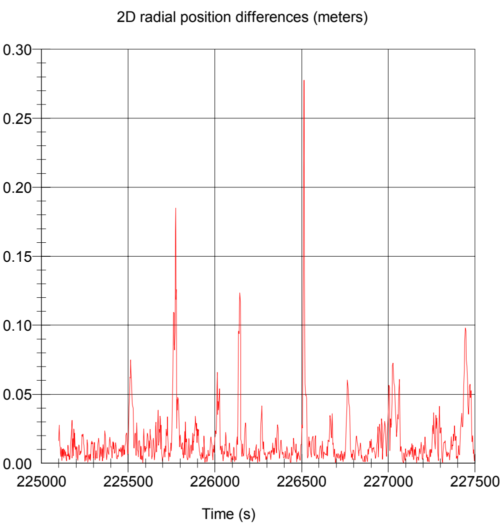

Figure 5: Radial position error with 10 second GPS outages

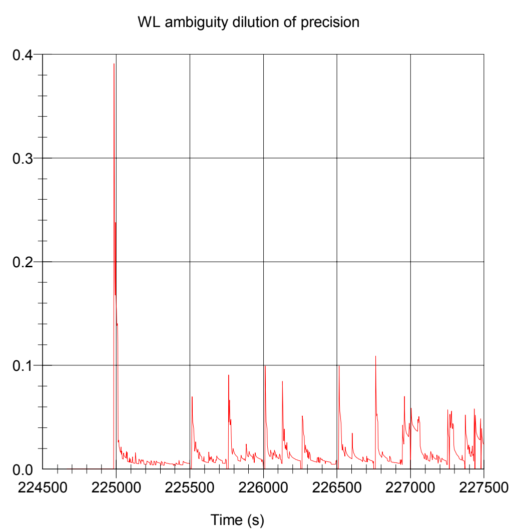

Figure 6: WL ADOP with 10 second GPS outages

Figure 6 and Figure 8 show the wide-lane ADOP with 10 and 300 second outages. With 10-second outages, inertial and DR aiding maintains the precision of the wide-lane ambiguities, whereas with 300-second outages there is no significant precision retention. This is consistent with the qualitative ADOP reduction described by (17).

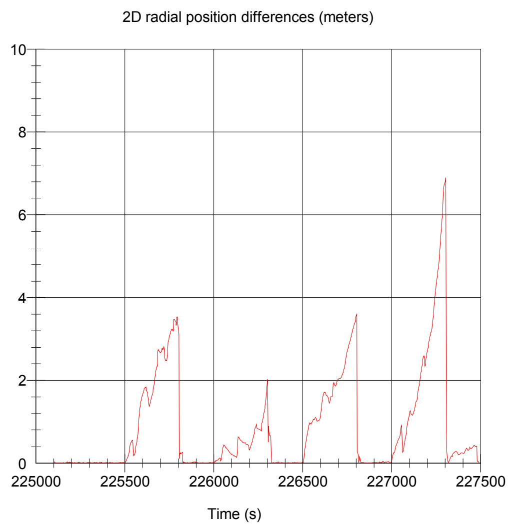

Figure 7: Radial position error with 300 second GPS outages

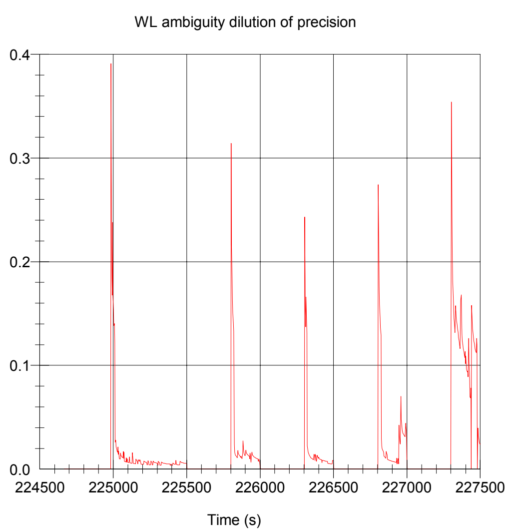

Figure 8: WL ADOP with 300 second GPS outages

SINGLE FREQUENCY TESTS

The previous dual frequency test data were repeated with access to the L2 observables disabled. Table 2 shows the position error standard deviations and times to L1 integer ambiguity resolution. The acronym *NR* designates no resolution before the start of the next scheduled outage. Reliable times-to-fix within 10 seconds were demonstrated following 10-second outages. These were in fact faster than for the dual frequency case because there is no time expended on wide-lane ambiguity resolution. Reliable times-to-fix within 30 seconds were demonstrated following up to 30-second outages. Thereafter ambiguity resolution within a reasonable wait time became unreliable.

Table 1: Dual frequency ambiguity resolution and positioning statistics

Outage Duration (seconds)	Radial Position Error RMS (meters)	Time-to-fix L1 ineger ambiguities (seconds)
10	0.1	8, 6, 5, 5, 7, 6
30	0.3	5, 7, 9, 6, 9, 9
60	0.8	9, 9, 7, 7, 7, 8
120	1.4	11, 8, 8, 6, 9, 9
180	2.0	9, 11, 7, 7, 8, 63
240	2.5	9, 9, 7, 39
300	3.0	9, 8, 13, 131
600	6.0	55, 13

Table 2: Single frequency ambiguity resolution and positioning statistics

Outage Duration (seconds)	Radial Position Error RMS (meters)	Time-to-fix L1 ineger ambiguities (seconds)
10	0.1	5, 6, 4, 2, 5, 4
30	0.3	6, 22, 5, 7, 14, 8
60	0.8	NR, 11, 5, 7, 143, NR
120	1.4	NR, NR, NR, 71, 75, 19

CONCLUSIONS

This paper has described an inertially aided RTK position and orientation system that implements tightly coupled GPS observables integration and tightly coupled ambiguity resolution through the use of a single Kalman filter that models INS errors and floated ambiguities. A covariance analysis of IARTK following a GPS outage and consequent loss of phase continuity examined potential floated ambiguity observability enhancement and equivalent search space volume reduction brought about by retained inertial position accuracy during the outage. The position error grows with time, so that in the limit of arbitrarily long outages there is no improvement in ambiguity resolution performance. The position error rate during the outage can be reduced with the use of DR aiding sensors such as a DMI.

This paper has examined the advantages of IARTK through simulated GPS outages in real van test data, and has shown that inertial aiding provides for significant improvement in the time-to-fix over unaided RTK. This is particularly true when only L1 phase data are made available to the IARTK system. These tests confirm the IARTK concept described by the analysis.

Inertially aided RTK provides a reliable and predictable improvement in time to recovery of integer RTK position accuracy following a GPS outage. Inclusion of DMI aiding constrains the position error rate and hence the growth of the ambiguity search space on larger outages and thereby provides a longer retention of floated ambiguity precision. An IARTK system using a relatively inexpensive "tactical grade" IMU provides reliable times-to-fix of 10 seconds or better following outages lasting up to 120 seconds. Thereafter the advantage of inertial aiding diminishes progressively.

This paper has examined inertially aided L1-only RTK positioning and demonstrated the same rapid integer RTK recovery following outages lasting up to 30 seconds. Inertial aiding therefore provides robust positioning at the decimeter level over short outages once the system has obtained initial fixed L1 ambiguities.

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Figure 9: POS/LV 320 used in the IARTK tests

Figure 10: Test vehicle for the IARTK tests